chick and Mason, 1961) fit the relationship $\Omega^{(1,1)*} = 1.452\ (T^*)^{-0.5}$ to within 1% over the range $T^* = 0.27$ to $T^* = 1.39$. Other functions have not been investigated.

A more detailed analysis (for which there is insufficient space here), with certain reasonable assumptions, shows that where indeterminacy occurs bounds exist on the values of σ and ϵ/k which might be obtained by fitting experimental values to theoretical equations. The bounds on the values of ϵ/k are determined by (and indeed are inversely proportional to) the bounds on T^* within which (6) holds. Thus any lower bound on T^* will determine an upper bound on ϵ/k and vice versa. The bounds on σ will be determined by the constancy of $\sigma^i(k/\epsilon)^L$. (Since both for viscosity and second virial coefficient, i and L are of opposite sign, for these two properties, a lower bound on T^* will determine a lower bound on σ and vice versa). In respect of viscosity (and probably the other properties, too), a region exists (at high temperature) for which there is only a lower bound on T^* (Liley, 1972).

Similarly, if the ratio of the highest absolute temperature covered by the experimental data to the lowest is insufficiently removed from unity, indeterminacy may still occur in practice even though the T^* values lie outside a range in which (6) holds closely. This is because the curvature in a plot of $\log|D^*|$ versus $\log T^*$ may be insufficient to overcome the effect of the narrowness of the experimental temperature range. (The numerical example of Martin mentioned above is an illustration of this point.)

This indeterminacy of the potential parameters for a given potential function should not be confused with the somewhat analogous phenomenon (occurring over a different though nearby temperature range) whereby various different potential functions can, by appropriate adjustments of their respective potential parameters, be made to predict indistinguishable y-T plots (Klein and Hanley, 1968; Smith et al., 1965).

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A Turbulent Flow Model for Mass Transfer in Wetted Wall Columns and Packing

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Emmert and Pigford (1954) studied gas absorption in falling liquid films. This study showed that a mass transfer model based upon a parabolic velocity distribution in the film compared favorably with absorption and desorption data in which the surface rippling was suppressed by wetting agents. Data for films with rippling show mass transfer rates that are several times greater than with rippling suppressed. Portalski (1963) presented and analyzed data for film thickness on a smooth, vertical plate with the conclusion that the data appeared to be best represented by a universal velocity profile treatment. This note presents a turbulent film model for comparison with mass transfer data with rippling present.

THEORY

A turbulent falling film can be assumed to be represented by a flat velocity profile with mass transfer to the film from the wall or from the gas corresponding to the penetration theory. This is expressed by the equation

$$k_L = 2\sqrt{\frac{D}{\pi\theta}} = 2\sqrt{\frac{Du}{\pi z}} \qquad (1)$$

where the time of contact θ is obtained from the average film velocity and the distance between transverse mixings. Substitution of $\Gamma/\rho y$ for the average film velocity and rearrangement to obtain the dimensionless mass transfer coefficient in terms of the Schmidt and Reynolds numbers yields

$$k_L{}^+ = \frac{(N_{Sc})^{-1/2}}{u^*}\left(\frac{4\Gamma}{\mu}\right)^{1/2}\frac{\nu}{\sqrt{\pi z y}} \qquad (2)$$

TABLE 1. CALCULATED CONTACT DISTANCES

Reference	System	N_{Sc} or N_{Pr}	$\frac{4\Gamma}{\mu}$	z, cm
Emmert and Pigford	O_2-air	360	300	1.2
Emmert and Pigford	O_2-air	360	990	1.6
Emmert and Pigford	CO_2-air	440	150	1.1
Emmert and Pigford	CO_2-air	440	1,000	1.4
Stirba and Hurt	Stearic acid-ethanol	4,500	665	1.4
Stirba and Hurt	Paraffin wax-butanol	18,000	376	1.3
Garwin and Kelly	Water	4.5	1,000	1.7

For a vertical falling film $u^* = \sqrt{gy}$, and from the universal velocity profile relationships $y = y^+\nu/u^*$.

$$k_L^+ = \frac{(N_{Sc})^{-1/2}}{\sqrt{\pi z}} \left(\frac{4\Gamma}{\mu}\right)^{1/2} \frac{\nu^{1/3}}{g^{1/6}y^{+2/3}} \tag{3}$$

The Portalski data for water can be represented by the equation

$$y^+ = 0.27 \left(\frac{4\Gamma}{\mu}\right)^{2/3}$$

for the Reynolds number range of 100 to 1000. Substitution in Equation (3) provides

$$k_L^+ = 1.34 \left(\frac{4\Gamma}{\mu}\right)^{1/18} \frac{\nu^{1/3}}{g^{1/6}z^{1/2}} (N_{Sc})^{-1/2} \tag{4}$$

Table 1 shows calculated values of the distance z for the oxygen and carbon dioxide absorption data of Emmert and Pigford. The mass transfer data of Stirba and Hurt (1955) for stearic acid-ethanol and paraffin wax-*n*-butanol and heat transfer for water reported by Garwin and Kelly (1955) are shown as examples of wall to film transfer. The calculated contact distances are observed to be about the same for transfer to the film from the gas and from the wall which indicates that the flat velocity profile is a reasonable assumption. The distances are also observed to correspond to the wave lengths reported for falling films by Tailby and Portalski (1962).

Stirba and Hurt also report mass transfer data for suppressed films. These data show agreement with the turbulent wall region equation for pipe flow (Hughmark, 1972)

$$k_L^+ = 0.065\ (N_{Sc})^{-2/3}$$

rather than Equation (4).

PACKING

Davidson et al. (1959) reported data for CO_2 absorption in water flowing over a string of spheres. Data from five and fifteen spheres are shown by Figure 1 in comparison to the wetted wall data of Emmert and Pigford. Dimensionless mass transfer coefficients are observed to be higher than for the wetted wall column and appear to increase with increasing Reynolds number.

Limited data are available for liquid phase mass transfer in packed columns. Danckwerts and Sharma (1966) summarized mass transfer coefficients and interfacial area data for carbon dioxide absorption with different packing. Figure 1 shows data for three different packings with the Reynolds number calculated for the experimental interfacial area and the film thickness corresponding to this Reynolds number for a wetted wall column. These dimensionless mass transfer coefficients are observed to be independent of Reynolds number and equal to the values for the wetted wall column for 1.27 cm Pall rings. The 2.54 cm packing shows an increase in mass transfer coefficient with increasing Reynolds number. This dimensionless correlation may prove useful when data become available for different liquid systems.

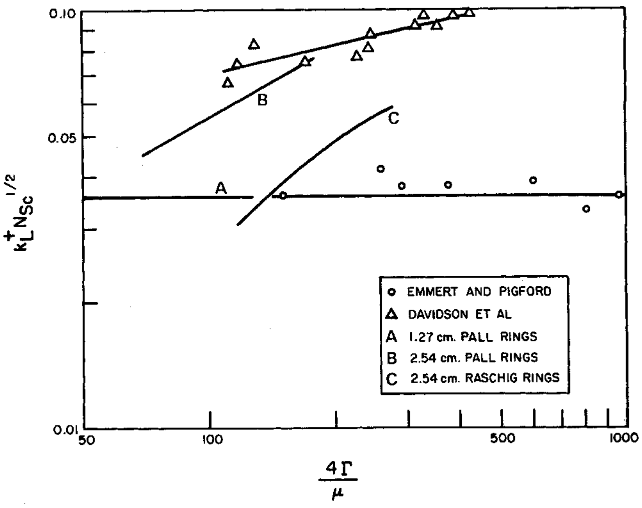

Fig. 1. Wetted wall and packing data.

NOTATION

D = molecular diffusivity
g = acceleration of gravity
k_L = liquid phase mass transfer coefficient
k_L^+ = dimensionless mass transfer coefficient, k_L/u^*
N_{Pr} = Prandtl number
N_{Sc} = Schmidt number
u = average film velocity
u^* = shear velocity
y = film thickness
y^+ = dimensionless film thickness
z = distance between transverse mixings

Greek Letters

Γ = liquid flow per unit periphery
μ = liquid viscosity
ν = liquid kinematic viscosity
ρ = liquid density
θ = contact time between transverse mixings

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